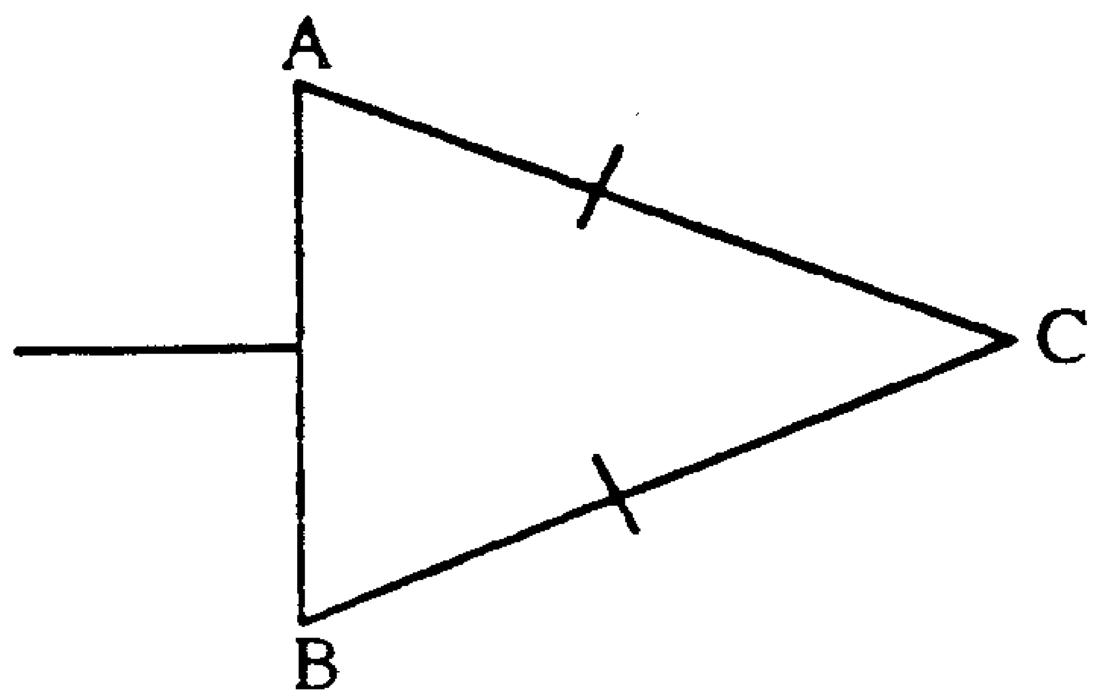
JRAHS 2U 1995

Question 1. (Start a new page.)

a) Given that P, Q, R are the points (1,3), (2,0), and (5,1) respectively :
(i) show that the line PQ is perpendicular to QR;
(ii) prove that the triangle PQR is isosceles;
(iii) find the coordinates of S, such that PQRS is a square.

b) A triangle LMN is right angled at L.
If $LM=x$, $NM=x+3$, and $NL=5$, find x.

c)



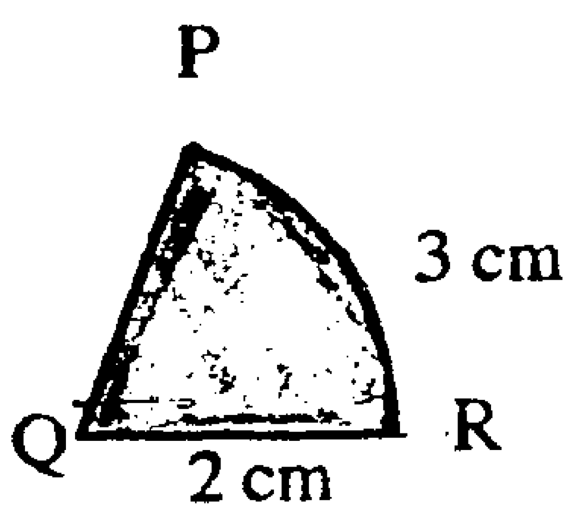
Find the exact area of the arrow head, where AC= BC = 6 cm and $\angle ACB = 45^\circ$.

d) Joan invested $5,000 at the beginning of 1985 at 8% per annum, compounded at the end of each year.
What will her investment be worth at the end of 1995 ?

Question 2. (Start a new page.)

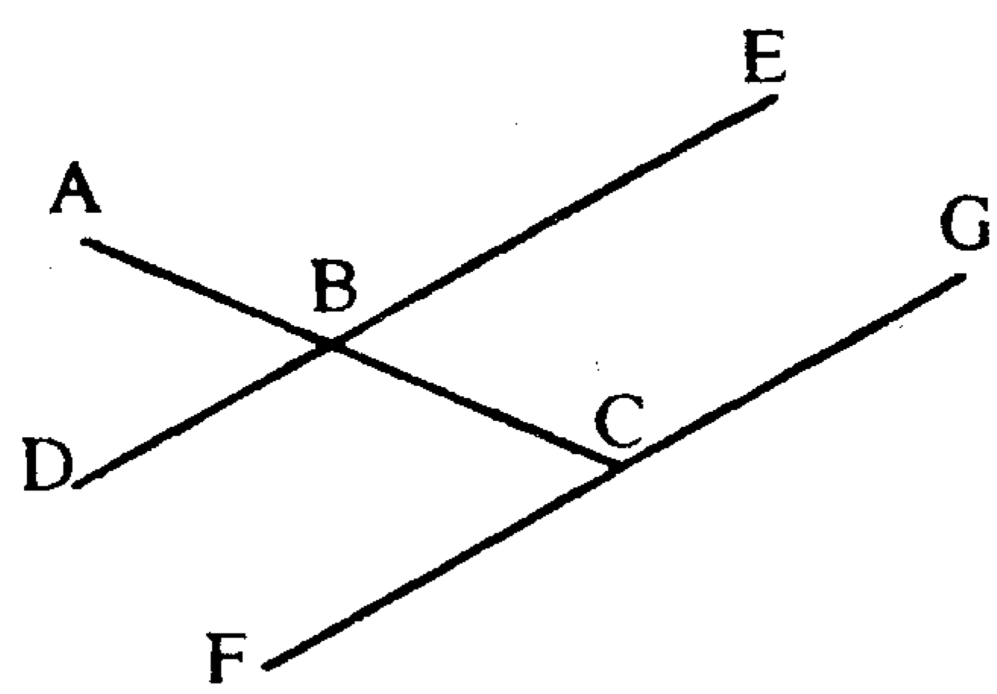
a) Differentiate: (i) x^2e^x (ii) $\ln(\cos x)$

b) The shaded figure, PQR is a sector of a circle, radius 2 cm.
Arc PR is 3 cm in length.



(i) Find angle $P\hat{Q}R$ in radians.
(ii) Find the area of the sector.

c) $\angle ABD = 47^\circ$ and $\angle BCG = 133^\circ$
Prove that lines DE and FG are parallel.



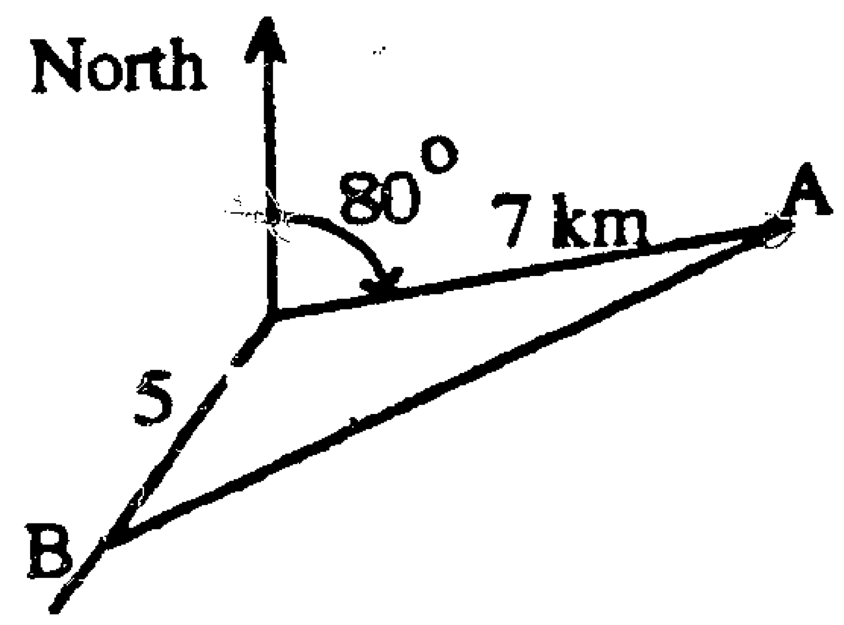
d) Simplify (i) $\sin(\pi - x)$ (ii) $\ln e^{3x}$

Question 3. (Start a new page.)

a) (i) Integrate $(4x+3)^5$.

(ii) Find the exact value of $\int_0^{\pi/6} \cos 2x\, dx$.

b) A ship A is 7 km away from a lighthouse L on a bearing of 080° and a ship B is 5 km away from the lighthouse on a bearing 210° as shown in the diagram.



(i) Find the distance between the ships A and B to the nearest km.

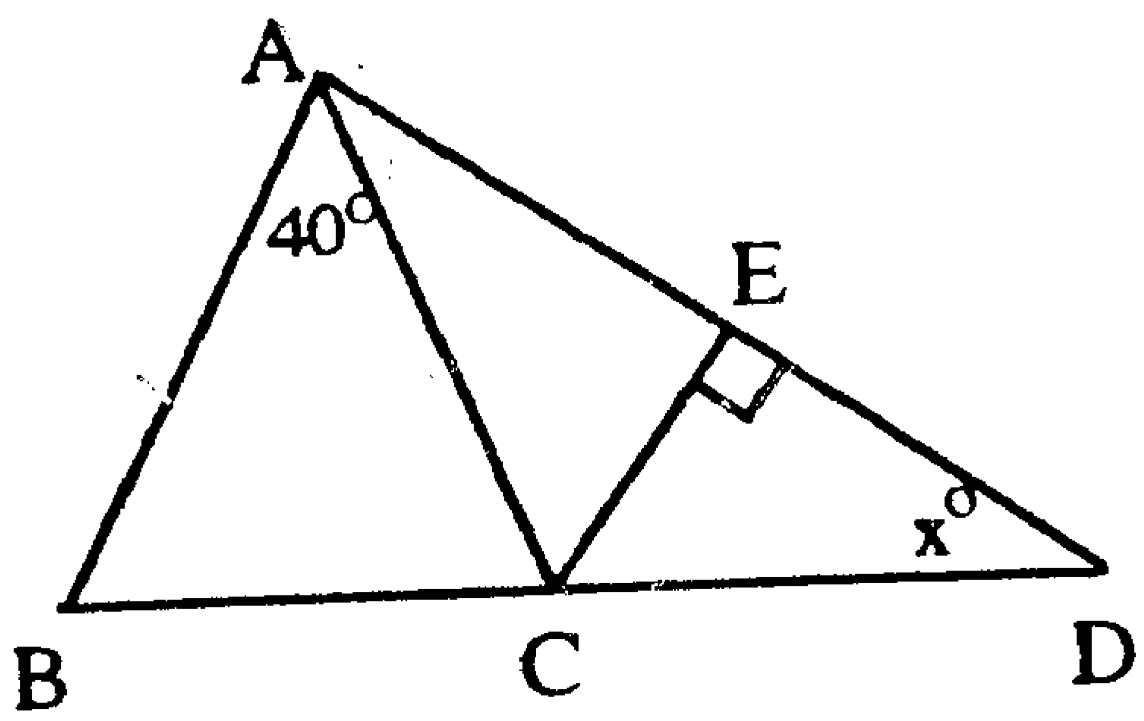
(ii) Find the bearing of the ship A from the ship B, to the nearest degree.

c) Solve $2\cos 2x = 1$ for $0 \le x \le 180$.

Question 4 (Start a new page.)

a)

(not to scale)



AB = AC
CE bisects ∠ACD
CE ⊥ AD.

(i) Find x°, giving reasons.

(ii) Prove that triangle ACD is isosceles.

b) The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 - 4$.

(i)(α) Find $\frac{d^2y}{dx^2}$

(β) Find the values of x for which the curve both increases and is concave downwards. Give reasons for your answer.

(ii) If the curve passes through the point (1, −2), find the equation of the curve.

Question 5 (Start a new page.)

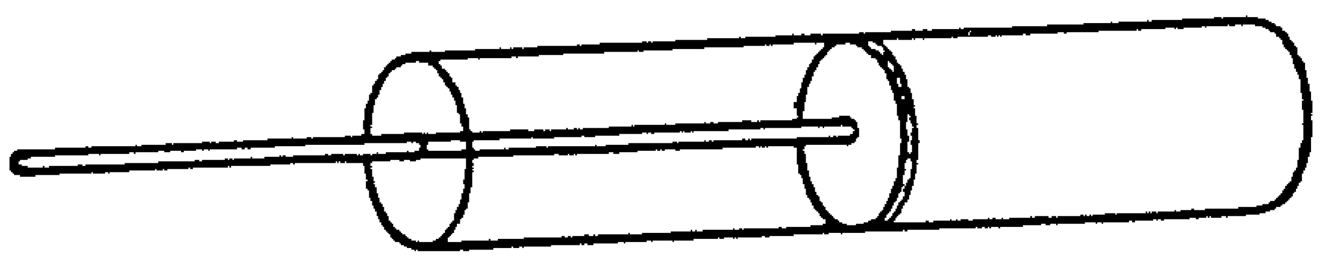
a) Consider the curve $y = 4x^3 - 3x^4$.
   (i) Find the points where the curve cuts the x-axis.
   (ii) Find any stationary points and determine their nature.
   (iii) Find all points of inflexion.
   (iv) Sketch the curve showing the above results.

b) Find the area defined by the curve $y = 4x^3 - 3x^4$ and the x-axis.

Question 6. (Start a new page.)

a) To start a game a player has to throw a 6 with a die.
Find the probability that the player starts at :
   (i) his first throw.
   (ii) his second throw.

b) The displacement of the piston shown, starting from the middle of the cylinder, is modelled by the equation $x(t) = \frac{1}{4}\sin(\pi t)$, where x is in metres and t in seconds.



   (i) Sketch the displacement-time function for the first 2 seconds.
   (ii) Calculate the total distance over which the piston moves in the first 5 seconds.
   (iii) Calculate the maximum speed of at which the piston moves.

c) A tree was 12 metres high at the start of a year and it increased by 1.5 metres during that year. If in each succeeding year the growth is $\frac{3}{4}$ of that during the previous year, find the limiting height.

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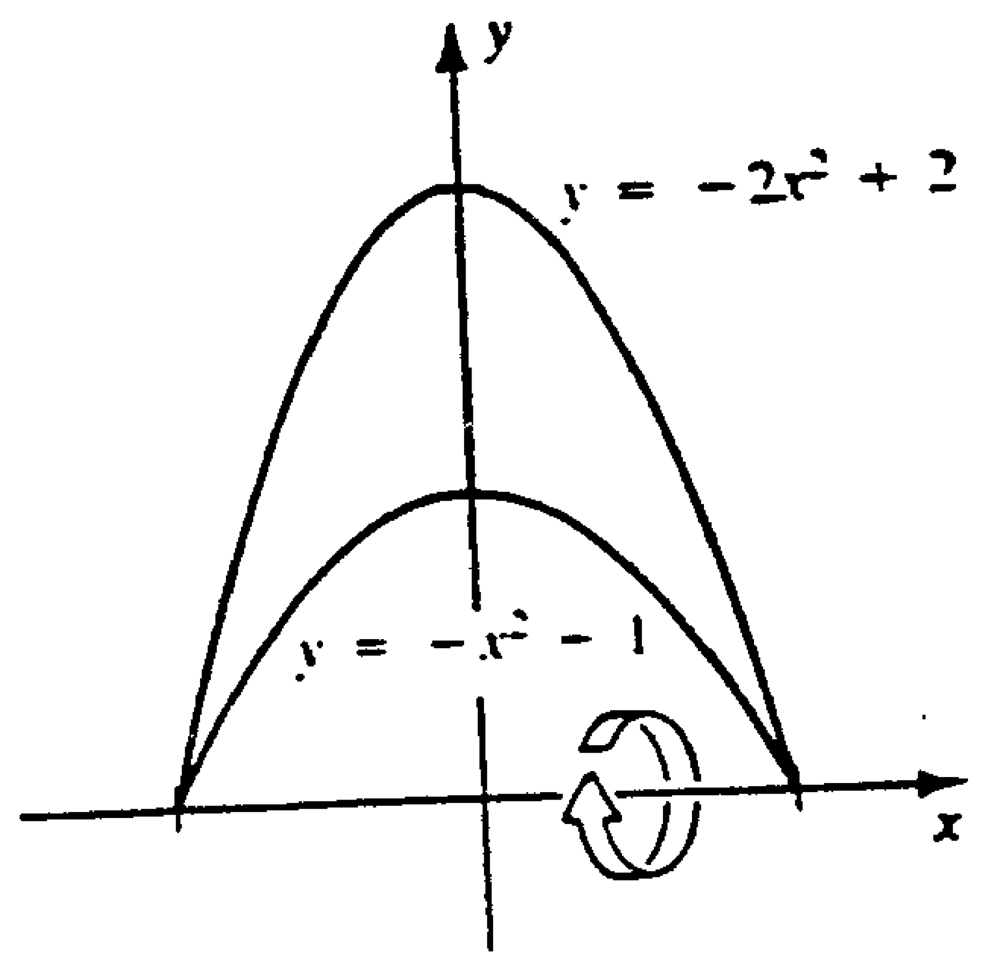
Question 7 (Start a new page.)

a) A body moves in a straight line. At time t seconds its acceleration is given by $a = 6t + 1$. At $t = 0$ the body is at the origin and its velocity is -2 m/s.

(i) Show that the velocity is given by $v = 3t^2 + t - 2$.
(ii) Determine when the particle is at rest.
(iii) Describe the motion.

b)



The area between the two parabolas $y = -2x^2 + 2$ and $y = -x^2 + 1$ is rotated about the x axis.

Find the volume of the solid thus generated.

Question 8 (Start a new page.)

a) (i) Simplify $\log_2 18 - 2\log_2 \sqrt{3}$.
(ii) Sketch the graph of $y = \ln x$.
(iii) Solve the equation $2\ln x - \ln(2x+3) = 0$.

b) A school is divided in two parts : Senior school with 400 boys and 200 girls and Junior school with 400 girls and 300 boys. A first student is chosen at random from the whole school. If this student comes from the Junior school, a second student is chosen from the Senior school; if the first student comes from the Senior school then a second student is chosen from the Junior school. By making use of a tree diagram, or otherwise, find the probability that :

(i) the second student chosen will be a girl;
(ii) if the second student chosen is a boy, he is from the senior school.

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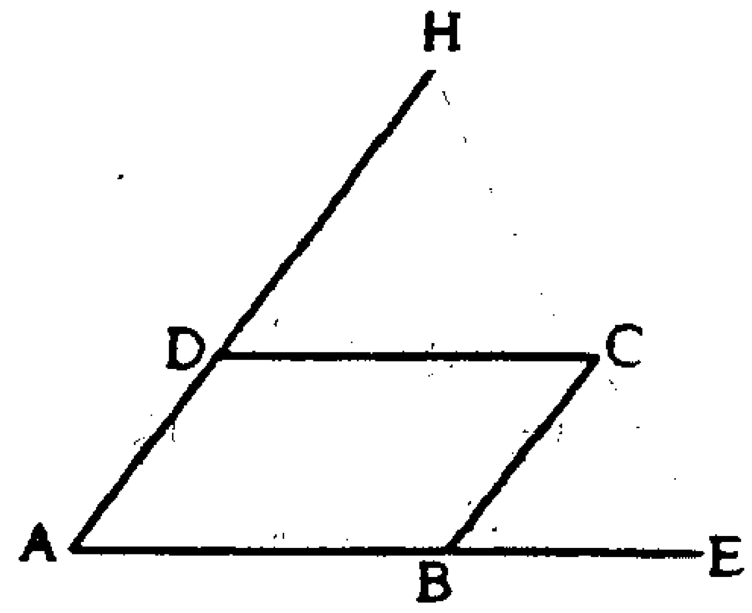
Question 9 (Start a new page.)

a) In a parallelogram ABCD the sides AB and AD are extended to E and H respectively so that $\dfrac{AD}{DH} = \dfrac{BE}{AB}$. Prove that :



(i) the triangles HDC and CBE are similar;

(ii) the points H, C and E lie on a straight line.

b) A prize fund is set up with a single investment of \$20,000, to provide an annual prize of \$1,500. The fund accrues interest at 5% per annum, paid yearly. If the first prize is awarded one year after the investment, find :

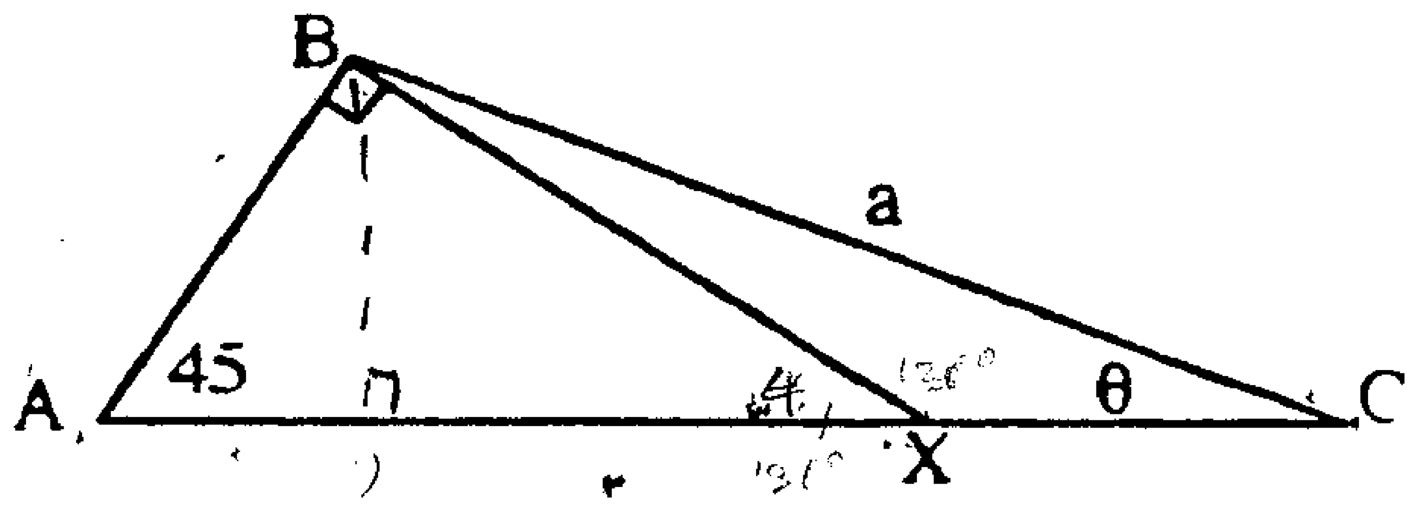
(i) the value of the prize fund immediately after the first prize has been awarded;

(ii) the number of years for which the full prize can be awarded.

Question 10 (Start a new page.)

a) (i) In ΔABC, BX is perpendicular to AB. Prove that $XC = a(\cos\theta - \sin\theta)$.

(ii) Hence find the exact value of XC when $a = 4$ cm and $\theta = 30^\circ$.



(not to scale)

b) One thousand trout, each one a year old, are introduced into a large pond. The number still alive after t years is predicted to be

$$N = 1000\,e^{-0.205t}.$$

(i) Show that the number of trout decreases at a rate proportional to the number of trout alive.

(ii) The weight $W(t)$ (in kg) of an individual trout is expected to increase according to the formula $W(t) = 0.1 + 0.5t$. After approximately how many years is the total weight (in kg) of all the trout in the pond a maximum ?

The End

J.R.A.H.S. 1995 2U TRIAL

Qu 1: (a) (i) $m_{PQ} \times m_{QR} = -1$

$-3 \times \frac{1}{3} = -1 \therefore PQ \perp QR$

(ii) $PQ = \sqrt{10}$, $QR = \sqrt{10}$ $\therefore \triangle PQR$ is isosceles

(iii) $S(4,4)$

(b) $x = \frac{8}{3}$

(c) $A = 9\sqrt{2}$

(d) \$11,658

Qu 2:

(a) (i) $xe^{x}(2+x)$

(ii) $-\tan x$

(b) (i) 1.5^c

(ii) 3cm^2

(c) —

(d) (i) $\sin x$

(ii) $3x$

Qu 3: (a) (i) $\frac{(4x+3)^6}{24} + c$

(ii) $\frac{\sqrt{3}}{4}$

(b) (i) 11kms

(ii) 59°

(c) $x = 30°, 150°$

Qu 4: (a) (i) $x = 35°$

(ii) —

(b) (i) $6x$

(ii) $x < -\frac{2\sqrt{3}}{3}$

(iii) $y = x^3 - 4x + 1$

Qu 5: (a) (i) $(0,0)$ $(\frac{4}{3}, 0)$

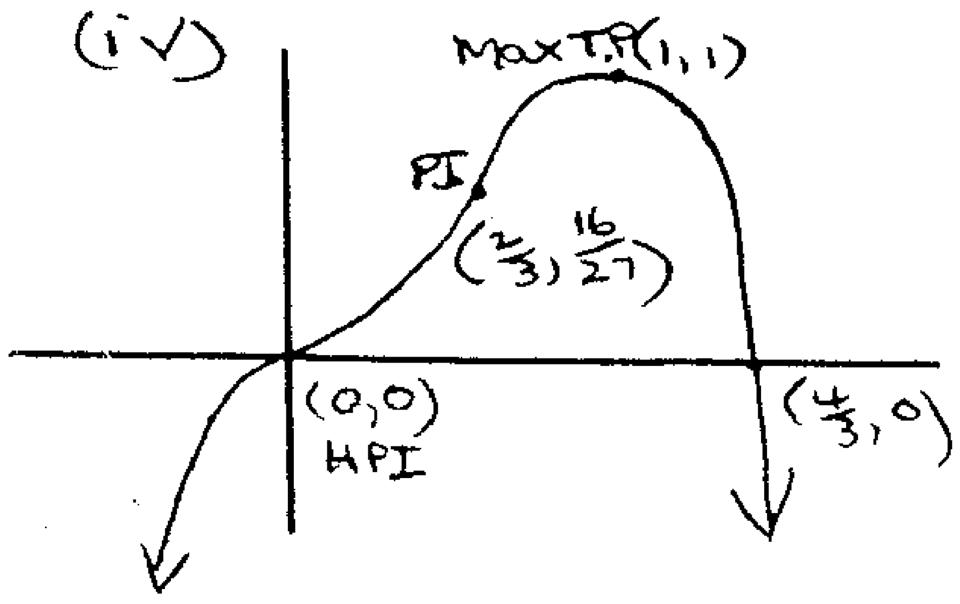
(ii) $(0,0)$ Horizontal P.I.

$(1,1)$ Max T.P.

(iii) $(0,0)$ already found

$(\frac{2}{3}, \frac{16}{27})$

(iv)

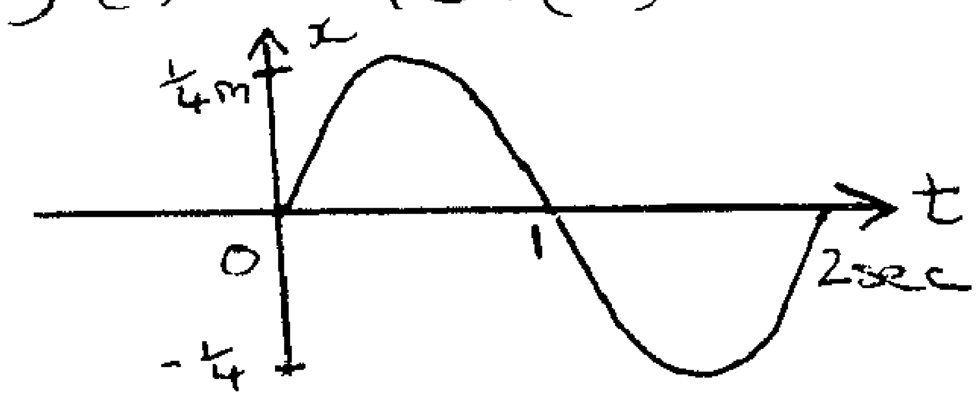


(b) $\frac{256}{405} u^2$

Qu 6: (a) (i) $\frac{1}{6}$

(ii) $\frac{5}{36}$

(b) (i) $x = \frac{1}{4}\sin(\pi t)$



(ii) $2\frac{1}{2}$m

(iii) $\frac{\pi}{4}$ m/s

(c) 18 m

Qu 7: (a) (i) —

(ii) $t = \frac{2}{3}$ sec

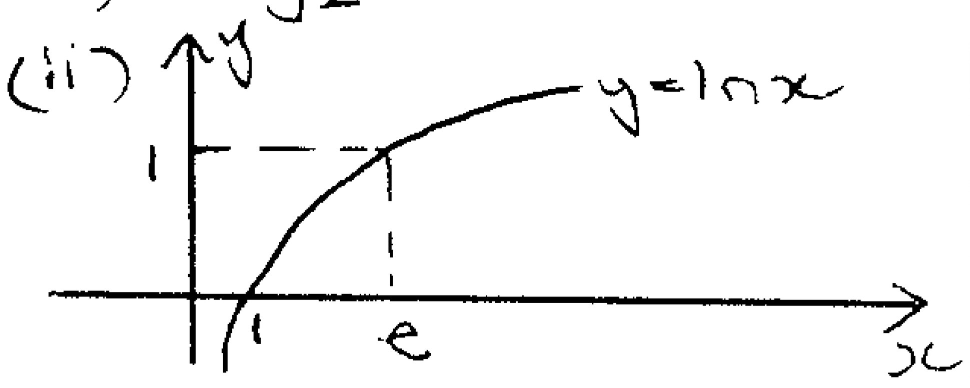
(iii) Initially particle is at the origin and moves to the left, slowing down (since force to the right opposing motion). Particle stops at $t = \frac{2}{3}$ sec and then changes direction moving now towards the right for

(b) $V = \frac{16\pi}{5} u^3$

Qu 8:

(a) (i) $\log_2 6$

(ii)



(iii) $x = 3$

(b) (i) $\frac{121}{273}$

(ii) $\frac{49}{76}$

Qu 9:

(a) (i) —

(ii) —

(b) (i) \$19,500

(ii) 22 yrs

Qu 10:

(a) (i) —

(ii) $2\sqrt{3} - 2$

(b) (i) $\frac{dN}{dt} = -0.205N$

(ii) 4.7 yrs.